LISBON SCHOOL OF ECONOMICS & MANAGEMENT

UNIVERSIDADE DE LISBOA

MASTER IN ACTUARIAL SCIENCE

Risk Models

29/01/2018

Time allowed: 3 hours

Instructions:

1. This paper contains **8** questions and comprises **4** pages including the title page.
2. Enter all requested details on the cover sheet.
3. You have 10 minutes of reading time. You must not start writing your answers until instructed to do so.
4. Number the pages of the paper where you are going to write your answers.
5. Attempt all questions.
6. Begin your answer to each of the questions on a new page.
7. Marks are shown in brackets. Total marks: 200.
8. Show calculations where appropriate.
9. An approved calculator may be used.
10. The distributed formulary and the Formulae and Tables for Actuarial Examinations (the 2002 edition) may be used. Note that the parametrization used for the different distributions is that of the distributed formulary.

1. For a portfolio of policies, you are given:

(i) The policy limit varies by policy.

(ii) A sample of ten payments is:

50 75 80 80 125 125+ 200+ 250 250+ 300

where the symbol + indicates that the loss exceeds the policy limit (i.e. 125+ means that the policy limit is 125 and the loss exceeds 125)

(iii) $S(x)$ is the survival function for the losses (not the payments)

a) **[10]** Using the Kaplan-Meier estimator, obtain a 95% confidence interval for $S(150)$ based on the direct method.

b) **[10]** Get another 95% confidence interval for $S(150)$ based on the log transformed method and the Nelson-Aalen estimator.

c) Now, assume that the losses follow an exponential distribution with unknown parameter θ.

i. **[10]** Obtain a maximum likelihood estimate for θ and for $S(150)$

ii. **[15]** Using the delta method obtain an approximation for the variance of $\hat{S}(150)$ and determine a 95% confidence interval for $S(150)$

d) **[15]** Now, assume that not only the losses follow an exponential distribution with unknown parameter θ but that you also know that policyholders number 4 and 6 have ordinary deductibles (losses smaller than the deductible are not reported and losses higher than the deductible are paid in excess of the deductible). The deductibles are 30 (for policyholder 4) and 10 (for policyholder 6). Show that the maximum likelihood estimate for θ is unchanged and explain why it happens.

2. **[15]** A random sample of 400 claim amounts (thousands of euros) originates the following results

Claim Size	$(0;0.25]$	$(0.25;0.5]$	$(0.5;1.0]$	$(1.0;2.0]$	$(2.0;5.0]$	$(5.0;10.0]$	over 10.0
Number of claims	20	40	**a**	130	60	**b**	10

You also know that, using the ogive, $F_n(0.95) = 0.3975$. Calculate **a** and **b** and give an estimate for the probability that a claim amount is between 0.95 and 1.5.

3. From a continuous population you are given the following sample

(1.0; 1.5; 1.5; 2.3; 2.3; 3.5; 3.5; 3.5; 5.0; 5.0).

The kernel has been defined as $k_y(x) = 0.75\left(1-(x-y)^2\right)$, $y-1 < x < y+1$.

a. **[10]** Calculate the kernel density estimate of $f(1.8)$.

b. **[10]** Prove that $k_y(x)$ fulfills the necessary conditions to be considered a kernel function.

4. Consider that the distribution function of the losses is $F(x \mid \alpha, \beta) = \dfrac{1}{1+(x/\beta)^{-\alpha}}$, $x > 0$, $\alpha, \beta > 0$.

a. **[5]** Show that the quantile of order g of this distribution is given by

$$q_g = \beta\left(\frac{g}{1-g}\right)^{1/\alpha}$$

b. **[10]** Having observed the sample

53.4	82.0	92.0	129.2	154.2	178.3	180.4	200.4
245.7	262.9	295.9	355.0	355.2	400.0	435.1	592.3
774.8	795.1	799.6	1232.9	1257.9	1782.7	2448.1	

obtain an estimate of the unknown parameters using the percentile matching method. Choose the 30th and 70th smoothed percentiles.

5. Now, assume that the sample given in question 4b was selected from a Gamma distribution with $\alpha = 3$, i.e. $f(x \mid \theta) = \dfrac{x^2\, e^{-x/\theta}}{2\theta^3}$, $x > 0$, $\theta > 0$. Remember that, for this case, $F(x \mid \theta) = 1 - e^{-x/\theta}\left(1 + \dfrac{x}{\theta} + \dfrac{x^2}{2\theta^2}\right)$, $x > 0$.

a. **[10]** Obtain a maximum likelihood estimate for θ and for $P(X > 250)$.

b. **[5]** Get an estimate for the variance of the maximum likelihood estimator of θ.

c. **[15]** Using the asymptotic distribution of the m.l.e., get a 95% confidence interval for θ. Can we get a better confidence interval? If your answer is yes, explain how.

6. A random sample $(x_1, x_2, \cdots, x_n)$ where $n = 40$, $\sum_{i=1}^{n} x_i = 20$ was observed from a Bernoulli population with parameter θ. Using a Bayesian point of view let us assume that the prior for θ is given by a beta distribution with parameters $\alpha = 6$ and $\beta = 2$, i.e. $\pi(\theta) = \frac{\Gamma(8)}{\Gamma(6)\Gamma(2)} \theta^5 (1-\theta)$, $0 < \theta < 1$.

a. **[10]** Obtain the posterior distribution for θ

b. **[5]** Obtain an estimate for θ based on a zero-one loss function and also obtain another estimate based on a quadratic loss function.

c. **[10]** Using Bayes central limit theorem determine a 95% HPD interval for θ

7. **[15]** You observed the following random sample (3.56; 4.28; 7.43; 15.34; 17.14). Test, using the Kolmogorov-Smirnov test, if it is acceptable ($\alpha = 0.05$) to consider that the corresponding population follows a Weibull distribution with parameters $\theta = 10$ and $\tau = 2$.

8. An approximation is needed for the distribution of the discounted value of the sum of 2 future payments. You know that the time to the first payment follows an exponential distribution with mean 0.2 and that the time between the two payments is also exponentially distributed with mean 0.3 (and independent of the time of the first payment). The amount of the first payment is 100 and the amount of the second payment is Weibull distributed with parameters $\tau = 2$ and $\theta = 100$. Assume that the discount factor is $v = 1/1.05$.

a. **[15]** Explain how to use simulation to get this approximate distribution and to estimate the probability that the discounted value is larger than 200.

b. **[5]** Assuming that the first generated random numbers are 0.2, 0.1, 0.8 obtain the first generated figure for the discounted sum.

SOLUTION

1.

a)

j	y_j	s_j	r_j	$\frac{(r_j - s_j)}{r_j}$	$\prod_{i=1}^{j} \frac{(r_i - s_i)}{r_i}$	$\frac{s_j}{r_j}$	$\sum_{i=1}^{j} \frac{s_i}{r_i}$
1	50	1	10	9/10	9/10	1/10	1/10
2	75	1	9	8/9	8/10	1/9	19/90
3	80	2	8	6/8	6/10	2/8	83/180
4	125	1	6	5/6	5/10	1/6	113/180
5	250	1	3	2/3	1/3	1/3	173/180
6	300	1	1	0	0	1	353/180

$$S_n(150) = 0.5$$

$$\hat{\text{var}}\, S_n(150) \approx 0.5^2 \left(\frac{1}{90} + \frac{1}{72} + \frac{2}{48} + \frac{1}{30} \right) = 0.025$$

The 95% confidence interval is given by $0.5 \pm 1.96\sqrt{0.025}$ i.e. $(0.1901; 0.8099)$

b)

$\tilde{H}(150) = \sum_{i:y_i \le 150} (s_i / r_i) = 113/180 = 0.6278$, then $\tilde{S}(150) = e^{-113/180} = 0.534$

To get the 95% log-transformed CI for $H(150)$

$$\tilde{\text{var}}\left(\tilde{H}(t)\right) = \left(\frac{1}{10^2} + \frac{1}{9^2} + \frac{2}{8^2} + \frac{1}{6^2} \right) = 0.0814$$

$$U = \exp\left(\frac{1.96\sqrt{0.0814}}{0.6278} \right) = 2.4369$$

Then the interval for $H(150)$ is $\left(\tilde{H}(150) \times (1/U); \tilde{H}(150) \times U\right)$, i.e. (0.2190;1.3008)

And for $S(150)$ we get (0.272; 0.803)

c)

Uncensored observations: $\ell_i(\theta) = -\ln\theta - \frac{x_i}{\theta}$ where x_i is the observed value

Censored observations: $\ell_i = -\frac{u_i}{\theta}$ where u_i is the censoring point

$$\ell(\theta) = -7\ln\theta - \frac{\sum x_i + \sum u_i}{\theta} = -7\ln\theta - \frac{1535}{\theta}$$

$$\ell'(\theta) = \frac{-7}{\theta} + \frac{1535}{\theta^2} \qquad \ell'(\theta) = 0 \Leftrightarrow \frac{7}{\theta} = \frac{1535}{\theta^2} \Leftrightarrow \theta = \frac{1535}{7} = 219.2857$$

$$\ell''(\theta) = \frac{7}{\theta^2} - \frac{1535 \times 2}{\theta^3}$$

$$\ell''\left(\frac{1535}{7}\right) = \frac{7}{(1535/7)^2} - \frac{1535 \times 2}{(1535/7)^3} = \frac{7^3}{1535^2} - \frac{7^3 \times 2}{1535^2} = -\frac{7^3}{1535^2} = -0.0001456 < 0$$

Then the m.l.e. for θ is $\hat{\theta} = 219.2857$ and for $S(150)$ is

$\hat{S}(150) = e^{-150/\hat{\theta}} = e^{-150/219.2857} = 0.5046$

Use the delta method to approximate the variance of $\hat{S}(150) = e^{-150/\hat{\theta}}$

$g(\theta) = e^{-150/\theta}$; $g'(\theta) = (150/\theta^2)e^{-150/\theta}$; $g'(\hat{\theta}) = (150/\hat{\theta}^2)e^{-150/\hat{\theta}} = 0.001574$

$\hat{\text{var}}(\hat{\theta}) \approx -1/(-0.0001456) = 6869.46$

$\hat{\text{var}}(g(\hat{\theta})) = \left(g'(\hat{\theta})\right)^2 \hat{\text{var}}(\hat{\theta}) = 0.01702$

Then determine the confidence interval for $S(150)$ Conf. interval for

$\hat{S}(150) \pm 1.96\sqrt{\hat{\text{var}}(\hat{S}(150)} = 0.5046 \pm 1.96\sqrt{0.01702}$ →(0.249; 0.760)

d)

Let x be the loss and y the corresponding payment.

Non-censored observations:

Without deductible, the contribution for the log likelihood is

$l_i(\theta) = \ln\left(\theta^{-1}e^{-x_i/\theta}\right) = -\ln\theta - x_i/\theta = -\ln\theta - y_i/\theta$ as $x_i = y_i$

With ordinary deductible d we have $x_i = y_i + d$ and

$$l_i(\theta) = \ln\left(\frac{\theta^{-1}e^{-x_i/\theta}}{e^{-d/\theta}}\right) = -\ln\theta - \frac{x_i}{\theta} + \frac{d}{\theta} = -\ln\theta - y_i/\theta$$

Censored observations:

Without deductible, the contribution for the log likelihood is

$l_i(\theta) = \ln\left(e^{-x_i/\theta}\right) = -x_i/\theta = -y_i/\theta$ as $x_i = y_i$

With ordinary deductible d we have $x_i = y_i + d$ and

$$l_i(\theta) = \ln\left(\frac{e^{-x_i/\theta}}{e^{-d/\theta}}\right) = -\frac{x_i}{\theta} + \frac{d}{\theta} = -y_i/\theta$$

And then we are maximizing the same function.

This is only true for the exponential distribution and this is due to the “lack of memory” property

2.

$$F_n(0.5) = \frac{60}{400};\ F_n(1.0) = \frac{60+a}{400}$$

$$F_n(0.95) = 0.3975 = \frac{0.05}{0.50}F_n(0.5) + \frac{0.45}{0.50}F_n(1.0) = 0.1\ \frac{60}{400} + 0.9\ \frac{60+a}{400}$$

Then $0.3975 \times 400 = 6 + 54 + 0.9a \Leftrightarrow a = 110$ and

$b = 400 - (20 + 40 + 110 + 130 + 60 + 10) = 30$

$\hat{P}(0.95 < X < 1.5) = F_n(1.5) - F_n(0.95)$

$F_n(1.5) = \frac{0.5}{1} F_n(1.0) + \frac{0.5}{1} F_n(2.0) = 0.5 \frac{170}{400} + 0.5 \frac{300}{400} = \frac{235}{400}$

Then $\hat{P}(0.95 < X < 1.5) = F_n(1.5) - F_n(0.95) = 0.5875 - 0.3975 = 0.19$

3.

a)

$$\hat{f}(1.8) = \sum_{j=1}^{k} p(y_j) \times k_{y_j}(1.8)$$
$$= 0.1 \times 0.75\left(1-(1.8-1.0)^2\right) + 0.2 \times 0.75\left(1-(1.8-1.5)^2\right) + 0.2 \times 0.75\left(1-(1.8-2.3)^2\right)$$
$$= 0.0270 + 0.1365 + 0.1125 = 0.276$$

x= 1.8

y_j	p(y_j)	k_y(1.8)	p(y_j)*k_y(1.8)
1.0	0.1	0.27	0.027
1.5	0.2	0.6825	0.1365
2.3	0.2	0.5625	0.1125
3.5	0.3	0	
5.0	0.2	0	
sum	**1**		**0.276**

b) We must show that $k_y(x) > 0$ and that $\int_{y-1}^{y+1} k_y(x)\,dx = 1$. Additionally we can prove that $\int_{y-1}^{y+1} x\,k_y(x)\,dx = y$ in order to guarantee that the expected value is preserved.

As $k_y(x) = 0.75 \times \left(1-(x-y)^2\right)$ for $y-1 < x < y+1$, i.e. $-1 < x-y < 1$ the first condition is obvious

$\int_{y-1}^{y+1} k_y(x)\,dx = \int_{y-1}^{y+1} 0.75 \times \left(1-(x-y)^2\right)dx = 0.75 \times \int_{-1}^{+1}\left(1-z^2\right)dz$, using $z = x - y$,

and

$$0.75 \times \int_{-1}^{+1}\left(1-z^2\right)dz = 0.75\left(z - z^3/3\right]_{-1}^{1} = 0.75\left((1-1/3)-(-1+1/3)\right) = 0.75 \times (2-2/3) = 1$$

Additionally it is easy to see that $k_y(x)$ is symmetric around y and consequently $\int_{y-1}^{y+1} x\,k_y(x)\,dx = y$

4

a)

The quantile q_g is the figure such that $F(q_g \mid \alpha, \beta) = g$.

As $F(x|\alpha,\beta)=\dfrac{1}{1+(x/\beta)^{-\alpha}}$ we get the equation to be solved in order to q_g.

The solution is

$$\frac{1}{1+(\mathrm{q}_g/\beta)^{-\alpha}}=g \Leftrightarrow 1=g+g\left(\frac{q_g}{\beta}\right)^{-\alpha} \Leftrightarrow \frac{1-g}{g}=\frac{\beta^\alpha}{q_g{}^\alpha} \Leftrightarrow q_g{}^\alpha=\frac{g\,\beta^\alpha}{1-g} \Leftrightarrow q_g=\beta\left(\frac{g}{1-g}\right)^{1/\alpha}$$

b)

Let's compute the empirical quantiles $\tilde{q}_{0.3}$ and $\tilde{q}_{0.7}$

$(n+1)\times 0.3=7.2$, then $\tilde{q}_{0.3}=0.8\times 180.4+0.2\times 200.4=184.4$

$(n+1)\times 0.7=16.8$, then $\tilde{q}_{0.7}=0.2\times 592.3+0.8\times 774.8=738.3$

The equations to be solved are

$$\begin{cases} q_{0.3}=\tilde{q}_{0.3} \\ q_{0.7}=\tilde{q}_{0.7} \end{cases} \Leftrightarrow \begin{cases} \beta\left(\dfrac{3}{7}\right)^{1/\alpha}=\tilde{q}_{0.3} \\ \beta\left(\dfrac{7}{3}\right)^{1/\alpha}=\tilde{q}_{0.7} \end{cases} \Leftrightarrow \begin{cases} \beta=\tilde{q}_{0.3}\,(7/3)^{1/\alpha} \\ \dfrac{\tilde{q}_{0.7}}{\tilde{q}_{0.3}}=\dfrac{(7/3)^{1/\alpha}}{(3/7)^{1/\alpha}} \end{cases}$$

Then

$$\frac{\tilde{q}_{0.7}}{\tilde{q}_{0.3}}=\frac{(7/3)^{1/\alpha}}{(3/7)^{1/\alpha}}=\left(\frac{49}{9}\right)^{1/\alpha} \Leftrightarrow \left(\frac{\tilde{q}_{0.7}}{\tilde{q}_{0.3}}\right)^{\alpha}=\frac{49}{9} \Leftrightarrow \alpha=\frac{\ln(49/9)}{\ln(\tilde{q}_{0.7}/\tilde{q}_{0.3})}=\frac{\ln(49/9)}{\ln(738.3/184.4)}=1.1716$$

and we get $\tilde{\alpha}=1.1716$ and $\tilde{\beta}=184.4\times(7/3)^{1/1.1716}=497.598$

5.

a.

$$f(x|\theta)=\frac{x^2\,e^{-x/\theta}}{2\theta^3},\ x>0,\ \theta>0$$

$$\ell(\theta)=\sum_{i=1}^n \ln\left(\frac{x_i^2\,e^{-x_i/\theta}}{2\theta^3}\right)=\sum_{i=1}^n\left(\ln(x_i^2)-(x_i/\theta)-\ln 2-3\ln\theta\right)$$

$$\ell'(\theta)=\sum_{i=1}^n\left(\frac{x_i}{\theta^2}-\frac{3}{\theta}\right)=\frac{\sum_{i=1}^n x_i}{\theta^2}-\frac{3n}{\theta}$$

$$\ell'(\theta)=0 \Leftrightarrow \frac{\sum_{i=1}^n x_i}{\theta^2}=\frac{3n}{\theta} \Leftrightarrow \theta=\frac{\sum_{i=1}^n x_i}{3n}=\frac{\bar{x}}{3}$$

$$\ell''(\theta)=-\frac{2n\bar{x}}{\theta^3}+3\frac{n}{\theta^2}$$

$$\ell''(\bar{x}/3)=\frac{1}{(\bar{x}/3)^2}\left(-\frac{2n\bar{x}}{\bar{x}/3}+3n\right)=\frac{-3n}{(\bar{x}/3)^2}<0$$

Then the m.l. estimate of θ is $\hat{\theta}=\dfrac{\bar{x}}{3}=\dfrac{569.7}{3}=189.9$

And for $P(X > 250)$ we get $\hat{P}(X > 250) = \mathrm{e}^{-250/\hat{\theta}}\left(1 + \frac{250}{\hat{\theta}} + \frac{250^2}{2\hat{\theta}^2}\right) = 0.8533$

b.

We can take advantage of the asymptotic variance of the mle and use

$\hat{\mathrm{var}}(\hat{\theta}) \approx -1/\ell''(\hat{\theta}) = \frac{\bar{x}^2}{27n} = 522.6378$

Or we can use the fact that X is gamma distributed with parameters $\alpha = 3$ and θ.

$\mathrm{var}(\hat{\theta}) = \mathrm{var}(\bar{X}/3) = \frac{\mathrm{var}(\bar{X})}{9} = \frac{\mathrm{var(X)}}{9n} = \frac{3\theta^2}{9n} = \frac{\theta^2}{3n}$ and then $\hat{\mathrm{var}}(\hat{\theta}) = \frac{\hat{\theta}^2}{3n} = 522.6378$

c.

A 95% confidence interval is given by $\hat{\theta} \pm 1.96\sqrt{\hat{\mathrm{var}}(\hat{\theta})}$, i.e. (145.09; 234.7)

Yes as, in this case, we know the exact distribution of the maximum likelihood estimator $\hat{\theta}$. In fact we can use $\frac{2\sum_{i=1}^n X_i}{\theta} = \frac{2n\bar{X}}{\theta} = \frac{6n\hat{\theta}}{\theta} \sim \chi^2_{(138)}$ as the pivotal quantity as $X_i \sim G(3,\theta)$, then $\sum_{i=1}^n X_i \sim G(3n,\theta)$ and finally $\frac{2\sum_{i=1}^n X_i}{\theta} \sim \chi^2_{(6n)}$.

Let q_1 and q_2 be the 0.025 and the 0.975 quantiles of a chi-square distribution with 138 degrees of freedom and we get the interval $\left(\frac{6n\hat{\theta}}{q_2}; \frac{6n\hat{\theta}}{q_1}\right)$, i.e. $\left(\frac{26206.2}{q_2}; \frac{26206.2}{q_1}\right)$ or (152.00; 244.07) after computing the quantiles

6

a.

$f(x \mid \theta) = \theta^x (1-\theta)^{1-x}$, $x = 0,1$, $0 < \theta < 1$

$L(\theta) = \prod_{i=}^n f(x_i \mid \theta) = \prod_{i=}^n \theta^{x_i}(1-\theta)^{1-x_i} = \theta^t (1-\theta)^{n-t}$ with $t = \sum_{i=1}^n x_i = 20$

$\pi(\theta) \propto \theta^5 (1-\theta) \qquad 0 < \theta < 1$

$\pi(\theta \mid x_1, \cdots, x_n) \propto L(\theta) \times \pi(\theta) = \theta^t (1-\theta)^{n-t}\, \theta^5 (1-\theta) = \theta^{5+t}(1-\theta)^{1+n-t}$

And then

$\theta \mid x_1, \cdots, x_n \sim beta(\alpha^* = 6 + t; \beta^* = 2 + n - t)$, i.e. $\theta \mid x_1, \cdots, x_n \sim beta(\alpha^* = 26; \beta^* = 22)$

b.

The zero-one estimate of θ is given by the mode of the posterior density, i.e.

$\pi(\theta \mid x_1, \mathrm{L}\ , x_n) \propto \theta^{5+t}(1-\theta)^{1+n-t}$

$\Psi(\theta) = \ln \pi(\theta \mid x_1, \mathrm{L}\ , x_n) = (5+t)\ln\theta + (1+n-t)\ln(1-\theta)$

$\Psi'(\theta) = \frac{5+t}{\theta} - \frac{1+n-t}{1-\theta} = \frac{15}{\theta} - \frac{11}{1-\theta}$

$$\Psi'(\theta)=0 \Leftrightarrow \frac{5+t}{\theta}=\frac{1+n-t}{1-\theta} \Leftrightarrow \frac{1-\theta}{\theta}=\frac{1+n-t}{5+t} \Leftrightarrow \frac{1}{\theta}=1+\frac{1+n-t}{5+t}=\frac{6+n}{5+t} \Leftrightarrow \theta=\frac{5+t}{6+n}=\frac{15}{26}$$

$$\Psi''(\theta)=-\frac{5+t}{\theta^2}-\frac{1+n-t}{\left(1-\theta\right)^2}=-\frac{15}{\theta^2}-\frac{11}{(1-\theta)^2}<0$$

Then the zero-one Bayes estimate for θ is $\theta^B_{0-1}=15/26=0.5769$

The estimate for θ based on a squared-loss function is the expected value of the posterior, $\theta^B_{SL}=E(\theta\,|\,x_1,\cdots,x_n)=26/48=0.5417$

c.

Using beta distribution:

A 95% approximate HPD can be defined by (l,u) such that l is the quantile 0.025 of the posterior and u is the quantile 0.975.

In practical terms one must solve two independent equations

$$\begin{cases} 0.025=\int_0^l \frac{\Gamma(48)}{\Gamma(26)\Gamma 22)}\theta^{25}(1-\theta)^{21}\,d\theta \\ 0.975=\int_0^u \frac{\Gamma(48)}{\Gamma(26)\Gamma 22)}\theta^{25}(1-\theta)^{21}\,d\theta \end{cases}$$

Using EXCEL one gets (0.4012; 0.6789)

Using Bayesian CLT

The posterior is approximated by a normal with mean $\frac{26}{48}=0.5417$ and variance $\frac{26\times 22}{48^2\times 49}=0.005067$ and then the HPD interval is given by $0.5417\pm 1.96\times\sqrt{0.005067}$, i.e. (0.4022; 0.6812)

7.

$H_0: X\sim F(x)=1-\exp(-(x/10)^2)\qquad H_1: H_0$ false

x_i	F(x_i)	Fn(x+)	Fn(x-)	max diff
3.56	0.119034	0.2	0	0.119034
4.28	0.167385	0.4	0.2	0.232615
7.43	0.424231	0.6	0.4	0.175769
15.34	0.904931	0.8	0.6	0.304931
17.14	0.947018	1	0.8	0.147018

D=0.3049

D(5,0.05)=1.36 $D_{5;'0.05}\approx 1.36/\sqrt{5}=0.6082$

So, we do not reject the null.

8.

a.

- Define NR, the number of replicas
- For each replica, $j = 1, 2, \ldots, NR$
  - Generate 2 exponentially distributed variables t_i, time to payment 1 and time between payment 1 and payment 2. To generate each t_i: generate u_i as a Uniform(0,1) random variable and use the inverse method, i.e. compute $t_i = -\theta_i \ln(1-u_i)$. Then $t_1 = -0.2\ln(1-u_1)$ and $t_2 = -0.3\ln(1-u_2)$
  - Generate a Weibull r.v., x, representing the amount of the 2nd payment. To generate each x: generate u_3 as a Uniform(0,1) random variable and use the inverse method, i.e. compute $x = 100\sqrt{-\ln(1-u_3)}$
  - Now compute the discounted value for replica j, y_j, and keep it.
    $y_j = 100 \times e^{-0.1t_1} + x \times e^{-0.1(t_1+t_2)}$
- The NR elements of array y are used to get an approximation of the required distribution (histogram, kernel density estimation, ….)
  To get an estimate the probability just count how many values of y_j are larger than 600 and divide this number by NR

b.

(the answer will depend of the chosen order to generate each variable)

$t_1 = -0.2\ln(0.8) = 0.044629$; $t_2 = -0.3\ln(0.9) = 0.031608$;
$x = 100\sqrt{-\ln(0.2)} = 126.8636$

$$y = 100e^{-0.004463} + 126.86e^{-(0.04463+0.03161)} = 99.55 + 125.90 = 225.45$$